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Ground-Based Implementation and Verification of Control Laws for Tethered Satellites

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Introduction

THE getaway tether experiment (GATE) is a single tether satellite system that will test and develop control technology for tethered system. Originally a free-flying tethered system released from a getaway special canister on the orbiter,[1] the system is now one experiment in the tether dynamics explorer (TDE) series launched by a Delta II.[2] The system consists of a Delta II second stage and subsatellite connected by a single tether. The subsatellite contains a motorized reel mechanism that will be the primary means of control actuation.[3]

A model of a single tether system that considered only length and in-plane libration dynamics was developed by Rupp[4] along with a tether tension control law requiring tether length/length rate feedback. Baker et al.[5] expanded this tension control law and performed simulations using a system model that included out-of-plane libration dynamics. Greene et al.[1] have applied a tension control law of the form proposed by Rupp to the GATE system. Simulation results demonstrated that the tension control law produced desirable results in the stabilization of the GATE system during deployment and retrieval maneuvers.

Modi and Misra[6] have developed extensive models of tethered systems and have performed simulations using control laws consisting of different forms of length-rate control. Recently, Davis and Banerjee[7] have expanded an idea introduced by Baker et al. in which out-of-plane librations are damped using a length-rate control after the tether has been deployed, or retrieved, to the desired length. A yo-yo type motion was used to perform out-of-plane libration damping requiring very high deployment and retrieval rates in relation to the frequency of the out-of-plane libration.

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The present work investigates the implementation of these control schemes to prototype hardware designed for space flight. A prototype reel mechanism has been constructed for use in the GATE system and ground tested. Both the out-of-plane libration control scheme proposed by Davis and Banerjee and a converted tension control law have been implemented with tether length and length rate available as feedback.

Prototype Design

The reel mechanism consists of a spool and a level wind mechanism that are driven by independent stepping motors. The spool is designed to hold 1.8 km of 0.075-cm-diam tether. The entire structure of dimensions 41.3 × 30.5 × 21.0 cm (16.25 × 12.0 × 8.25 in.) fits in the TDE experimental envelope. The mechanism is shown in Fig. 1.

Figure 2 is a simplified block diagram of the control system. Two stepping motors are used to eliminate any mechanical connections between the level wind mechanism and the spool shaft. Each motor driver requires a pulse frequency-modulated (PFM) signal to command the desired stepping rate. The stepper motor coupled to the level wind is electronically geared down by dividing the PFM signal frequency by three. Optical switches are used to trigger logic circuits providing the level wind motor with direction signals based on the spool motor direction and present level wind motor direction. An optical encoder connected to the spool shaft provides length and length-rate feedback. Pulses from the encoder produce a count that can be related to instantaneous tether length. The encoder signals also provide a direction of rotation signal, via logic circuitry, which is used as an up/down signal to the counter circuitry.

An IBM PC is used to control the reel mechanism. Interrupt-based control software was written in C language. At each interrupt, the tether length and length rate are calculated and new commands for the motors are formulated before interrupts are re-enabled. The motor commands consist of a motor step rate and direction of shaft rotation. The motor step-rate commands are transformed into a count word that is used by an 8253 timer to produce the PFM signal required by the motor driver.

System Equations

For testing purposes, the prototype was configured such that the tether is deployed toward the Earth. A mass of 22 g was placed on the end of the tether to produce a static tension in the tether, which is slightly larger than the static tension the actual system will experience when fully deployed.[2] In such a configuration, the equations of motion governing the pendu-

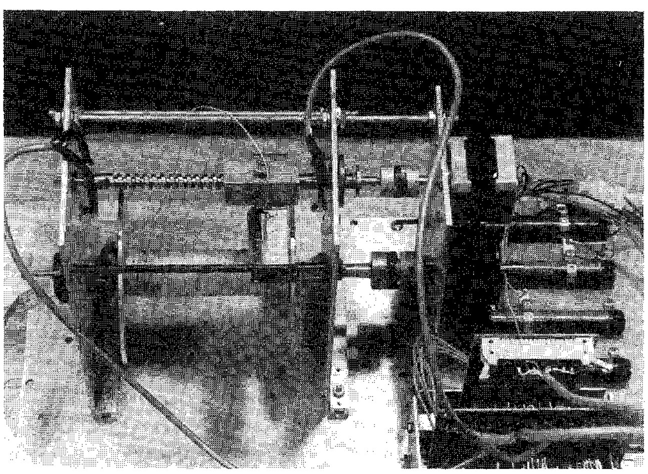

Fig. 1 Photograph of the reel/deployer mechanism. Note the two stepper motors: one is used to drive the reel while the other drives a spooling mechanism for level wind.

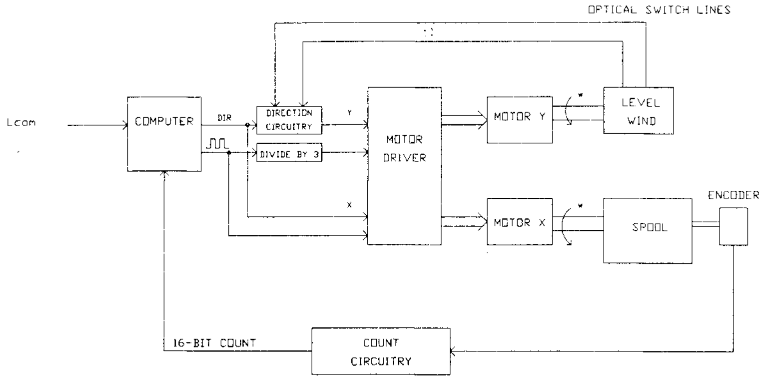

Fig. 2 Block diagram of the reel/deployer mechanism.

Table 1 Libration damping results

Test	Time to damp, s	Initial/final amplitude, deg
No control applied	53	7.0/1.3
Yo-yo simulation	43	7.0/1.3
Prototype with yo-yo	41	7.0/1.3
Prototype with phasing	36	7.0/1.3

lum system are analogous to those of the system model used by Rupp[4] and Baker et al.,[5] if air damping is ignored. Using a Lagrangian formulation, the equations are

$$\ddot{l} = g\cos\theta + l\dot{\theta}^2 - T/m \tag{1}$$

$$\ddot{\theta} = -(g\sin\theta)/l - (2\dot{l}\dot{\theta})/l \tag{2}$$

where l is the tether length, θ is either in- or out-of-plane angle, and T is input tension. Because of the absence of orbital rate, no cross coupling exists between the two angles. However, the angle dynamics of the prototype in the testing configuration are very similar to the out-of-plane equation used by Baker et al.[5]

Testing and Results

With the tether deployed to a length of 10 m, a pendulum motion was induced to test the out-of-plane libration control.[7] Since a tether angle was not measured in the prototype, this angle was estimated a priori and the required deployments and retrieval times were stored in a lookup table. A change in the tether length of 5% of the initial length was for both tether retrieval and deployment with a maximum rate of approximately 94 cm/s in both directions. The control law used is summarized as

$$L_{\text{command}} = L_{\text{set point}}\,(1 \pm 0.05) \tag{3}$$

where the decrease in commanded length occurs at $\theta_{\max}$ and $\theta_{\min}$ and the increase occurs at $\theta = 0$. Results from the computer simulation and the hardware implementation are given in Table 1.

With no control, the pendulum was damped by air drag and friction in 53 s. The prototype also outperformed simulation results. This improvement probably was due to unmodeled friction. Even better results were obtained by beginning each retrieval slightly before the maximum angle was reached and each deployment slightly before zero angle (phasing).

The results indicate that the prototype reel system should be able to perform the control scheme properly in the space-based system. In the space-based system, the orbital altitude will be at least 297 n.mi. (550 km). At this altitude, the out-of-plane oscillations would have a period of about 2882 s.[5] The maximum tether length rate of 94 cm/s produced by the prototype is more than sufficient to implement almost instantaneous changes in tether length with respect to such a long period of oscillation and remain within acceleration bounds preventing a slack tether condition described by Davis and Banerjee.[7]

Tether tension control of the form proposed by Rupp[4] was implemented by converting the control law to a length rate control law. Two different methods of converting the tension control law were developed. The first method was developed using knowledge of the tether length dynamics in the space-based system. The result was an equation for tether length acceleration which was integrated to produce a tether length-rate command. Since a desired tether length trajectory is produced, the method was called the trajectory method. The acceleration equation used is

$$\ddot{l} = l(\omega + \dot{\theta})^2 - T/m \tag{4}$$

where T is the tether tension commanded. The implementation of this method requires in-plane angle feedback or estimation.

The second method assumes that tension feedback is available (either measured or estimated), and an error signal is produced by differencing the desired tension and the actual tension. The error is used to produce a change in tether deployment rate that accelerates the tether and induces tether tension. This method was called the differencing method.

Short deployments using both methods were performed in simulation to obtain the desired response that stabilizes the in-plane angle. Deployments then were performed with the prototype using tether angle rate and tension estimation to provide the required feedback. The results are shown in Figs. 3 and 4. Each figure gives the length response as measured by the feedback encoder for the prototype. Figure 3 gives the

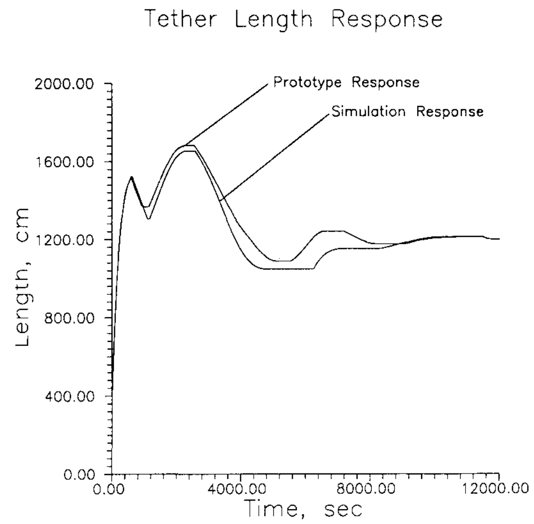

Fig. 3 Length response vs time for a 12-m deployment using the differencing method.

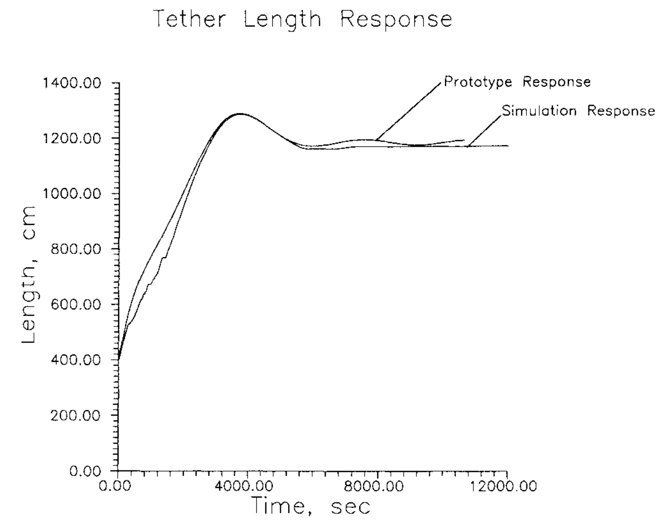

Fig. 4 Length response vs time for a 12-m deployment using the trajectory method.

results for the differencing method, while Fig. 4 shows the trajectory method results. Both methods produce tether length profiles comparable to simulation results. Maximum deployed length errors in both tests were approximately 8% for the trajectory method and less than 0.8% for the differencing method. Although the magnitude of either length error might be considered acceptable, the differencing method is clearly superior.

Conclusions

The results reported here indicate that the prototype reel mechanism performs satisfactorily compared to simulations. The demonstrated deployment and retrieval rates also appear adequate for a space-based system. Additionally, length errors were within acceptable limits and the yo-yo damping scheme has been shown successful.

Acknowledgment

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Estimating Damping in Higher Order Dynamic Systems

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Introduction

THOSE whose activities in engineering practice bring them into contact with dynamic systems are often concerned with determining the amount of damping present in a particular system. A common example is that of the control systems engineer tuning the forward path gain of a feedback control loop to optimize the transient response of one or more variables in the loop. Another is the experimental determination of modal damping coefficients of flexible systems. Existing experimental methods for obtaining quantitative damping data are lengthy and often require the introduction of unacceptable disturbances into the system.

This Note shows how finding just two points on a Bode diagram phase angle curve can, with remarkable accuracy, pinpoint the damping ratio of any system characterized by a dominant pair of complex poles. The experimental equipment required consists of a signal generator, a dual channel oscilloscope, and suitable transducers and other interface instrumentation. The procedure is straightforward and requires a minimum of supporting analysis.

Systems containing static and dynamic nonlinearities exhibit an approximate linear response when excited by small amplitude disturbances.[1] This allows such systems to be tested according to the method and their damping characteristics specified in approximate terms as a linear damping ratio, thus extending the utility of the method to many real systems.

Second-Order System

The frequency response of a second-order system is highly dependent on its damping ratio.[2] The classical family of Bode magnitude and phase curves depicted in any standard text on vibrations, dynamic systems, or control theory illustrates this dependency. It is seen that the slope of the phase curve at the point (ω_n, -90 deg) varies over a wide range from the critically damped ($\zeta = 1$) to the undamped ($\zeta = 0$). We know that the slope of a first-order system phase curve at the natural frequency ω_n is -66 deg/decade,[3] giving twice that value, or -132 deg/decade, for a critically damped second-order system at the undamped natural frequency. This slope becomes increasingly negative as damping is reduced, approaching infinity as all damping vanishes. The slope is thus seen to be a sensitive indicator of the amount of damping present, and a quantitative correlation of the slope with the damping ratio will allow the estimation of damping in real systems, both second order and those with a single pair of dominant complex poles.

Phase Angle Slope

The generalized second-order system transfer function

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1}$$

transforms to the frequency transfer function

$$G(j\omega) = \frac{\omega_n^4 - \omega_n\omega^2 - j(2\zeta\omega_n^3\omega)}{\omega_n^4 + (4\zeta^2 - 2)\omega_n^2\omega^2 + \omega^4} \tag{2}$$

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